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Estimación del asiento elástico producido por la excavación de un túnel mediante un método de los elementos finitos
Estimation of the elastic settlement produced by a tunnel excavation using finite element method

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Resumen-- **El trabajo que se presenta comprende el modelado mediante un método de los elementos finitos (en adelante MEF) para estimar el asiento elástico producido por la excavación de un túnel en un medio granular poroso sobre un estrato supuesto perfectamente impermeable. A su vez, sobre el estrato impermeable, el citado medio poroso se divide en dos estratos con diferente grado de saturación. En contacto con la capa superior se encuentran edificios aportando una carga superficial conjunta de 0,10 MPa. A una cierta profundidad y a cierta distancia horizontal respecto de la cimentación, se proyecta la excavación del túnel de 10 metros de diámetro. Se modela un FEM 2D con el empleo de las herramientas GMSH y MATLAB®. Se calculan los desplazamientos ocurridos en el terreno entre el estado inicial y el final. Para el cálculo de asientos se ha implementado en el MEF el principio de Terzaghi.**

Palabras clave— método de los elementos finitos; medio poroso; diseño de túneles; Principio de Terzaghi.

Abstract— **The work presented includes a finite element method (hereinafter FEM) to estimate the elastic settlement produced by the excavation of a tunnel in a porous granular medium on a supposed perfectly impermeable stratum. In turn, on the impermeable layer, the porous medium is divided into two layers with different degrees of saturation. In contact with the upper layer there are buildings providing a joint surface load of 0.10 MPa. At a certain depth and at a certain horizontal distance from the foundation, the excavation of the tunnel of 10 meters in diameter is projected. A 2D FEM is modelled using the GMSH and MATLAB® tools. The displacements occurred in the terrain between the initial and the final state are calculated. Terzaghi's principle has been implemented in the MEF for the calculation of settlements.**

Index Terms— finite element method; porous media; tunnel design; Terzaghi's Principle.

I. INTRODUCTION

This work includes the estimation of the elastic seat produced by the excavation of a tunnel with a discrete numerical formulation of the problem. The solution of the search displacement field is performed with a finite element moment (FEM). The settlement produced by tunnelling is a matter of interest in building engineering and architecture (Boscardin *et al.,* 1989; Bai *et al.,* 2014; Zhou *et al.,* 2016). Possible causes include vibrations and loss of bearing capacity (Tian *et al.,* 2019; Rallu *et al.,* 2023). But they are also based on the variation of effective pressures according to Terzaghi's postulate: "the resistance to shear stress and the change in volume of a soil depend on the magnitude of the effective pressure and its variations" (Terzaghi *et al.,* 1996).

The quantification of the maximum seat sought can be performed with analytical formulations. In this work, the bases and modeling by means of an MEF in a tunnel problem context

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in granular soils are exposed. The construction of the tunnel produces an infiltration into the interior of the tunnel and the corresponding variation in the effective stresses. The defined MEF numerically approximates the solution of the differential equations resulting from integrating Darcy's law with the continuity equation in a continuous porous medium (Zienkiewicz *et al.,* 1996; Zienkiewicz *et al.,* 2013; Gonzalez *et al.,* 2017; González *et al.,* 2007). The lower contour in contact with the saturated soil layers is an impermeable rock layer, constituting a de facto biphasic medium (Arduino, 1996). The problem is initially dynamic (Wang *et al.,* 2021). It is hypothesized that the flow within the tunnel is completely stabilized at a constant flow rate (Su *et al.,* 2017; Nikvar Hassani *et al.,* 2016). The chosen tunnel diameter is 10 m. The scope of this work is limited to an estimation of the displacements in certain nodes of a surface foundation, and to the key and floor of the tunnel, following a simplified process with situation before and after the construction of the tunnel. However, for estimates that require an accurate analysis of the entire tunnel outline, outside the scope of this work, a three-state procedure is required (Rodríguez *et al.,* 2023). For the case study, the analysis carried out allows a first estimation of the seat including the effect due to Terzaghi's principle

II. Methods

A. Fundamentals of the Finite Element Method in a Porous Media

The MEF used is traditional, with a formulation in displacements and isoperimetric Lagrangian quadratic finite elements (Olivella *et al.,* 2002; Liu *et al.,* 2022). However, due to the medium to which it is applied, which is a porous medium, the mathematical apparatus requires certain specific adaptations. The main variables of the stationary problem in porous medium are the piezometric head, the flow velocity, the hydraulic gradient or "motor" of the movement, and the interstitial pressure. As for the equations of governance of the problem, expressed in a field problem:

a) A mass conservation equation that must consider the saturated medium in which it is located. Considering that the stationary problem is addressed, once the flow is stabilized, you have:

$$\nabla q = \rho \cdot C \cdot \frac{\partial \emptyset}{\partial t} \qquad (1)$$

Where $\emptyset$ the total potential, which in our case is the piezometric height; ∇q The Conductive Term, ρ Density and C is the slope of the water storage curve. In the stationary problem, knowing that the velocity vector of the flow is, the problem is reduced to$\vec{v}$:

$$\nabla \vec{v} = -\frac{\partial h}{\partial t} = 0 \qquad (2)$$

b) Behavioural (or constitutive) law. In this kind of problem it is Darcy's law:

$$\vec{v} = -K\nabla h = K \cdot \vec{\iota} \qquad (3)$$

Where K is the coefficient of soil permeability, or hydraulic conductivity and the hydraulic gradient or "motor" of the motion and is equal to $\vec{\iota}$.

$$\vec{\iota} = -\nabla h.$$

By not considering heat transfer phenomena, Fourier's law can be dispensed with, so in the problem the integration of both equations provides the only equation that governs the problem: Laplace's equation:

$$\nabla^2 h = 0 \qquad (4)$$

The weak form of the MEF is used, with the weighted residuals method and the corresponding Galerkin approximation (the same form functions are used as used for the problem unknown) (Wang *et al.,* 2016; Korsawe *et al.,* 2006). Formulation in a porous medium, in a general case, requires integrating the Navier–Stokes equations. Darcy's law applies to the stationary problem (Larese *et al.,* 2015). And in the case analysed, assuming a stabilized flow, the problem is quite simplified with the Laplace equation. Gauss's divergence theorem simplifies the formulation for an MEF applied to a field problem (Zienkiewicz *et al.,* 2005; 2013). Applying the principle of virtual works (hereinafter PTV), Gauss's divergence theorem and the corresponding Galerkin approximation, we arrive at the following general expression of the MEF in porous medium expressed in residual form:

$$\nabla\omega = \int_{\Omega} \nabla\omega(K\nabla h)d\Omega - \oint_{\Gamma} \omega \cdot v_n d\Gamma = 0 \qquad (5)$$

Where v_n is the normal flow vector to the contour, Γ the contour, Ω the domain y $\nabla\omega$ the weighting function in the PTV. The negative sign in the second term is due to the flow following a direction from the highest to the lowest piezometric points.

As for the boundary conditions:

a) Essential boundary conditions or Dirichlet (Hansbo *et al.,* 2015): the problem in porous medium is to know the piezometric height h in one part Γ_h from contour. In practice, it involves knowing the water table: $h = \bar{h}$

b) Natural or Neumann's boundary conditions (Sandström *et al.,* 2013): in our case, they consist of knowing the derivative of the fluid flow through a part of the contour Γ_{v_n}. The problem of flow in porous medium usually refers to a contour condition with impermeability, as for example in our case soil-impermeable rock interaction: $\vec{v} \cdot \vec{n} = \bar{0}$

B. Implementation of Terzaghi's Principle

To implement Terzaghi's principle already expressed above, a sequential process in the MEF encoded in MATLAB ® is required. Terzaghi's principle, in its formulation, uses a material constant, the volumetric modulus of deformation K_T, which has the well-known formulation:

$$K_T = \frac{E}{3(1-2\nu)} \qquad (6)$$

Where E is the modulus of elasticity of the soil and the Poisson coefficient. The obtaining of seats by Terzaghi's principle is obtained by the following expression, knowing that the elastic model predicts that volumetric deformation is only produced by variations of normal octahedral stress (Paris, 1998; Berrocal, 2004). Therefore, tangential stresses produce only distortions. The νxy components of the total stresses before and after tunnel construction are not affected by interstitial pressures (Figure 9). For the general case:

$$\Delta\varepsilon_{vol} = \Delta\varepsilon_x + \Delta\varepsilon_y + \Delta\varepsilon_z = \frac{1-2\nu}{E}\left(\Delta\hat{\sigma}_x + \Delta\hat{\sigma}_y + \Delta\hat{\sigma}_z\right) \qquad (7)$$

So:

$$\Delta\varepsilon_{vol} = \frac{1}{K_T}\Delta\hat{\sigma}_{oct} \qquad (8)$$

Where $\Delta\hat{\sigma}_{oct}$ is the variation of the octahedral effective stress tensor and the variation of the volumetric strain tensor. The expression has the character of a constitutive law specific to Terzaghi's principle. For the case analysed, it is solved matrixial in two dimensions as an elastic problem together with the problem of flow in porous medium. An initial pore index for predominantly granular layers of 0.3 is considered to have a linear evolution with respect to effective stresses and there is no porosity reduction limitation for the latter. For the purposes of the calculation of the incremental seat, non-edometric conditions in planar deformation are considered.

$$\Delta\varepsilon_{vol}(\Delta\varepsilon_x \neq 0, \Delta\varepsilon_y \neq 0, \Delta\varepsilon_z = 0).$$

C. Case study

The case study refers to the type of section in Fig. 1. As for the variants of the problem, of interest for discussion, they are the following:

- *Situation prior to the construction of the tunnel.* The water table assumes a perfectly horizontal equipotential surface in accordance with an aquifer without inputs. The medium is porous with constant and identical permeabilities at x and y. This case is necessary to generate the matrix of effective pressures before the construction of the tunnel.

- *Tunnel terrain without waterproof coating.* There is no coating or support to prevent infiltration or deformation and displacement of the nodes in the tunnel contour. There could be light or semi-curdled shoring, but, in any case, the tunnel is deformable, and leakage is not prevented. We work with the hypothesis that infiltrations are evacuated from the tunnel immediately. This case is necessary to generate the matrix of effective pressures after tunnel construction.

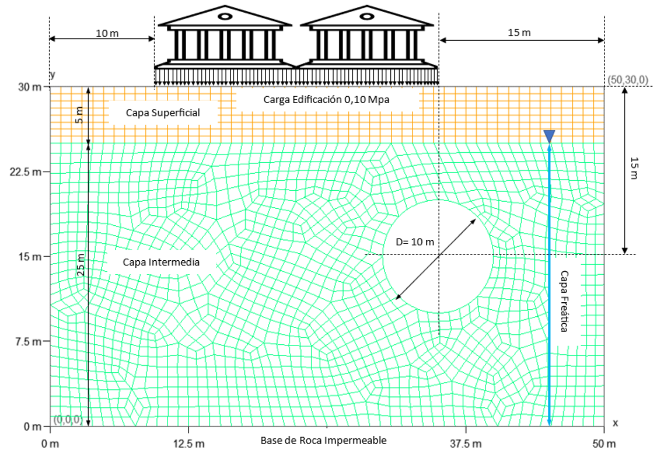

Fig. 1: Case study. Scheme. (Source: Authors' own creation)

- *Terrain with coated tunnel.* There is a perfectly rigid assumed support that prevents the deformation and displacement of the nodes in the tunnel contour.

- *Unlined and completely waterlogged tunnel terrain.* Leaks completely flood the tunnel.

- *Terrain with a coated and waterlogged tunnel.* Leaks completely flood the tunnel. In this case, work is carried out only in the state prior to any displacement, no matter how small of the tunnel contour.

The hypotheses for lined tunnels will be: perfectly rigid support, so there is no possible translation or rotation in the nodes of the contour that are supposed to be fixed in space. These cases have been selected to illustrate tunnels of old constructions, with very rigid masonry or very thick concrete coatings, perfectly seated and without any appreciable movement. In recently constructed tunnels it is not possible to apply the hypotheses of this case.

III. Results and discussion

After applying a code in MATLAB® with the relevant routines for the FEM problem posed and the case study, the following results are obtained. The approach is in displacements with isoparametric quadratic Lagrangian elements of 9 nodes. The meshing was carried out with Gmsh using the Frontal-Delaunay algorithm for high-quality meshes with rectangular elements. These results include matrices of

TABLE I
CASE STUDY. VARIABLES TO STUDY.

Type of Terrain per Layer	Depth	Density (kg/m^3)	K (m/s)	Degree of Saturation	E (Mpa)	ν	External building loads (Mpa/m/m.l.)
Superficial. Filling of various materials	0-5 m	1834,9	1·10-3	Humid unsaturated	40·106	0,3	0.10
Predominantly granular intermediate. Mixture of gravel, sand, and some silt and clay	5-30 m	1936,8	1·10-3	Saturated	40·106	0,3	0

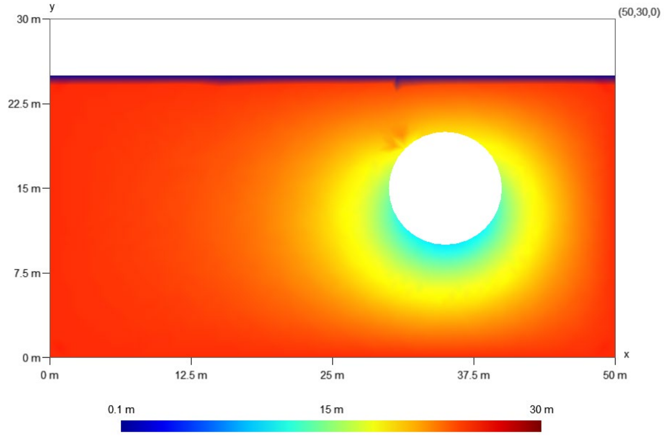

Fig. 2: Piezometric height. With tunnel. (Source: Authors' own creation)

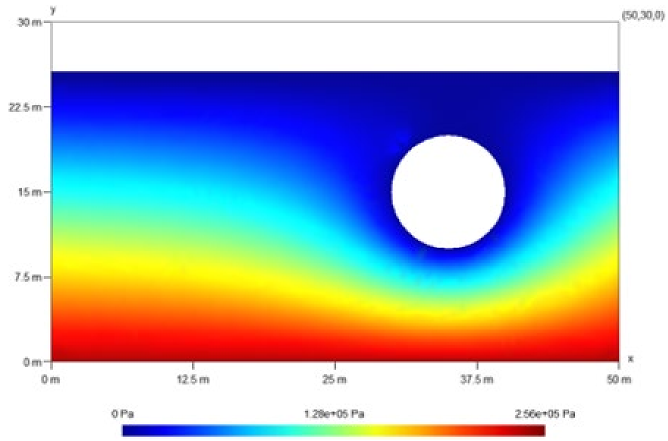

Fig. 3: Interstitial pressures. With tunnel. (Source: Authors' own creation)

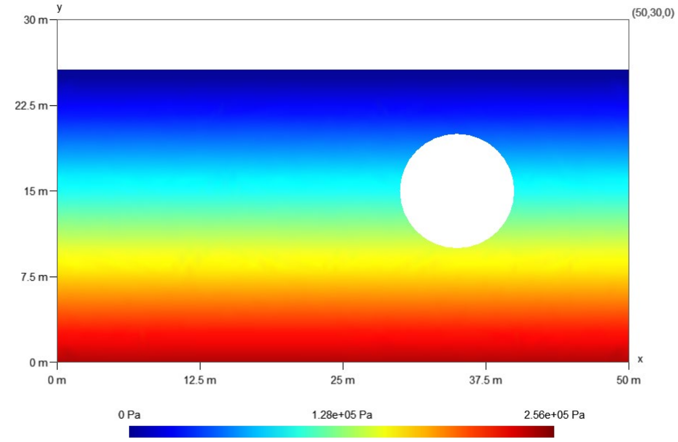

Fig. 4: Interstitial pressures. With a completely flooded tunnel. (Source: Authors' own creation)

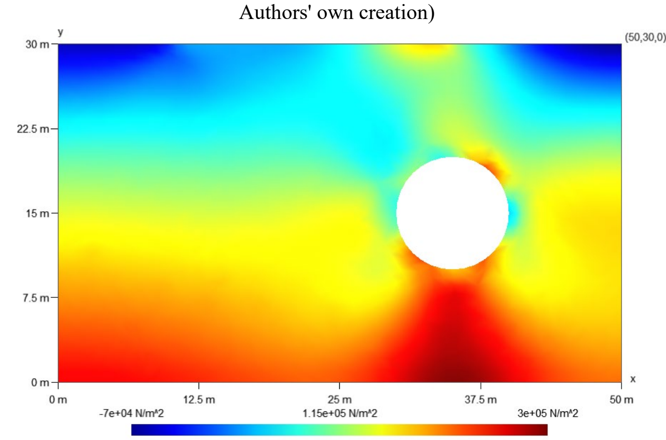

Fig. 5: Total stresses σ_xx. With tunnel. (Source: Authors' own creation)

numerical results relative to the different elements and knots of the mesh, as well as graphically interpretable results. The former have been used to define tables 2 to 4, which reflect the seats sought, while the latter will serve to visualize the effect of the tunnel construction on Fig. 2-17. The approach given to the exhibition is with the aim of illustrating the effect of Terzaghi's principle

A. Interstitial Pressure Field

To be able to determine the field of effective stresses, the determination of the field of interstitial pressures is required beforehand. Overall, the following sequence has been followed:

1) Determination of piezometric heights throughout the saturated domain.
2) Determination of the interstitial pressures resulting from the saturated medium.
3) Determination of total stresses according to an elastic problem (a constituent matrix for planar strain has been selected).
4) By the difference between the total stresses and the interstitial pressures, determination of the effective stress field sought.

The presence of flow in a saturated medium due to a tunnel will generate a gradient that will alter the usual horizontal equipotential surfaces in an isotropic, homogeneous porous medium with a horizontal surface as one of its boundary conditions. The determination of the piezometric height field can then be carried out in the MEF, assembling the elementary permeabilities, assumed in our case for an isotropic medium (equal permeability in all 3 directions). Interstitial pressures, a function of piezometric heights, will also be altered. By comparing the heights for cases with and without a tunnel, it is possible to see how an alteration occurs with the appearance of a hydraulic gradient.

Fig. 3 shows how it affects the tunnel, being inside at atmospheric pressure and, therefore, having the points of its contour a piezometric height equal to its geometric height.

Similarly, when transforming hydraulic loads to interstitial pressures in the granular porous medium of the case analysed, a clearer alteration is observed due to the tunnel effect. As expected, in the case of a completely flooded tunnel, the equipotential surfaces – once full and with stationary flow – will remain perfectly horizontal, just as if there were no tunnel.

B. Effective Stress Field

Then, to be able to calculate and present the field of effective stresses, it is necessary to calculate the total stresses beforehand. These stresses depend on the elastic characteristics of the materials and the effective pressures. Fig. 5 shows σ_{xx} and Fig. 6 σ_{yy}.

It is already possible to intuit the frequent failure of the tunnel floor in loose soils when the total stresses are observed.

This ruling, in which the tunnel "rises", is likely to occur in granular soils, among others, and is included, for example, in Circular Order 4/2007 of the Ministry of Public Works by obliging the concreting of the slabs of certain tunnels in certain contexts (Dirección General de Ferrocarriles, 2007). For the case without a tunnel and making use of surfaces of equal tension, the following voltage distribution is obtained σ_{yy} where you can see perfectly how the effect of the surface loads is felt in depth. The MEF makes it possible to detect the effect of surface loads at 30 m on the rock.

Fig. 8 illustrates how a rigid lining affects a perfectly seated tunnel, such as old subway galleries. In this case, you can see the effect produced by this tunnel, increasing the rigidity in its surroundings, and considerably altering the yy stress field. This effect, in practice, may involve increased settlements in areas close to (but not above or contiguous to) the tunnel. A typical case in urban areas is the construction of buildings on excavated soils and subsequently filled in next to relatively superficial old Metro works, with rigid masonry cladding on well-established soils (such as, for example, colossal works of old Metro galleries). Paradoxically, the buildings on the Metro will settle less than in the vicinity of the old work, generating dangerous differential settlements in the horizontal xx plane . Regarding shear stresses (Fig. 9), τ_{xy} are now commented.

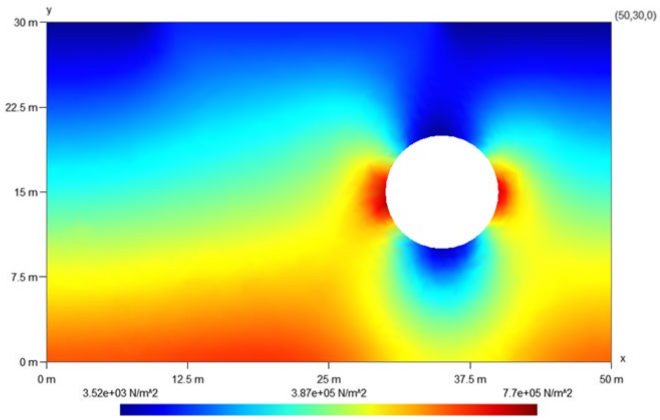

Fig. 6: Total Stresses σ_{yy}. With tunnel. (Source: Authors' own creation)

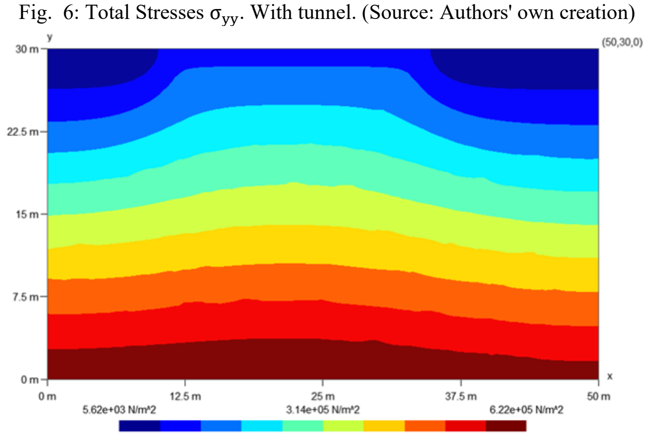

Fig. 7: Total stresses σ_{yy}. No tunnel. (Authors' own creation)

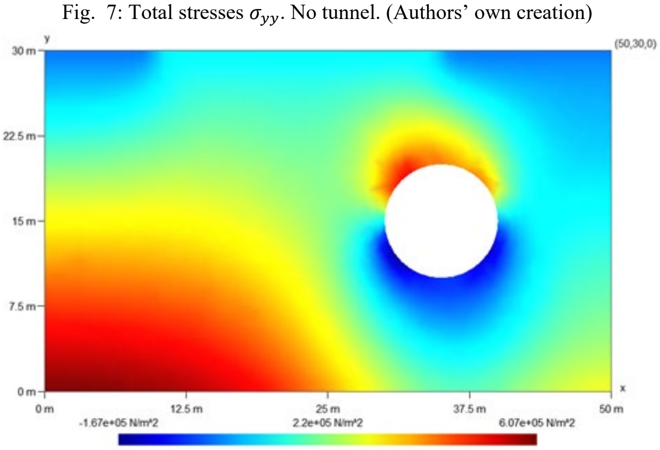

Fig. 8: Total stresses σ_{yy}. With lined and seated tunnel. (Source: Authors' own creation)

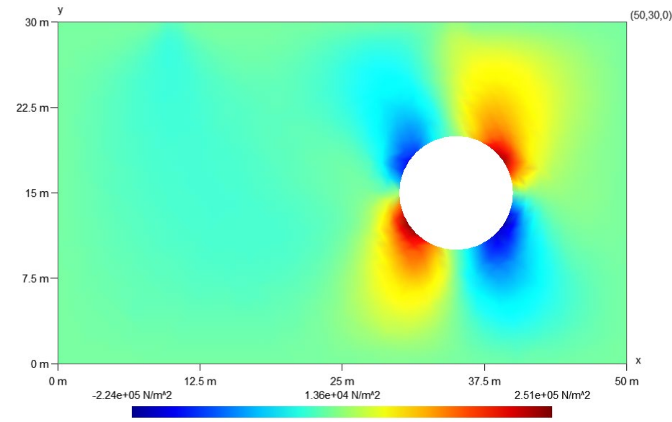

Fig. 9: Total stresses τ_{xy}. With tunnel.. (Source: Authors' own creation)

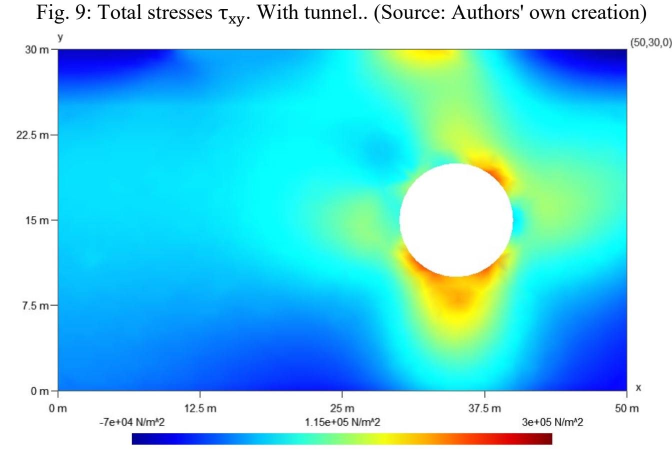

Fig. 10: Effective stresses $\hat{\sigma}_{xx}$. With tunnel. (Source: Authors' own creation)

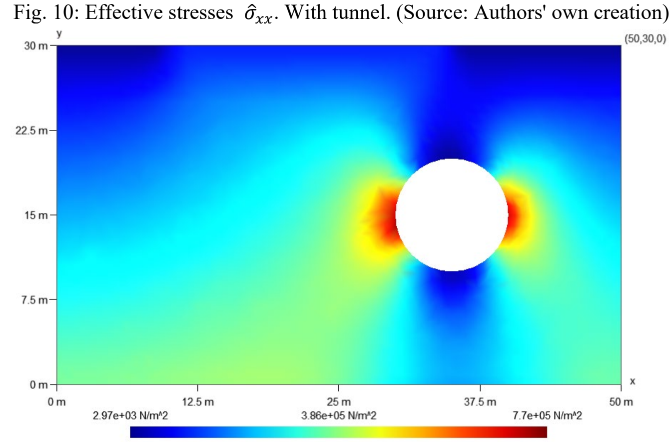

Fig. 11: Effective Stresses $\hat{\sigma}_{yy}$. With tunnel. (Source: Authors' own creation)

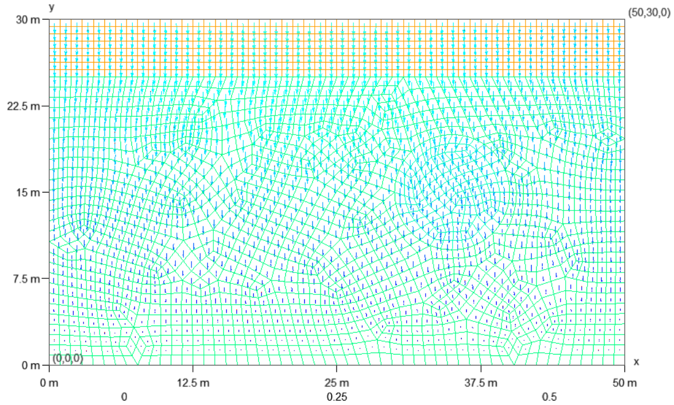

Fig. 12: Vertical displacement field (x5). Before the tunnel. (Source: Authors' own creation)

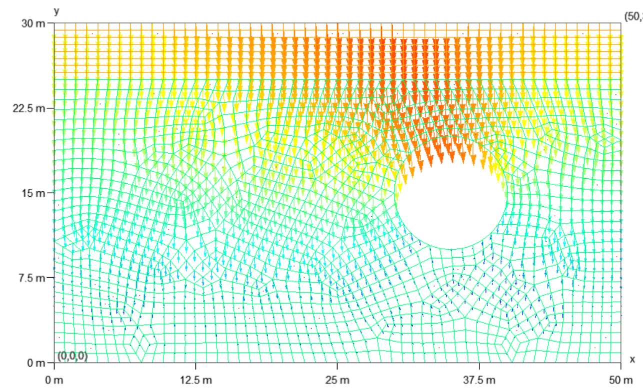

Fig. 13: Vertical displacement field (x5). With tunnel. (Source: Authors' own creation)

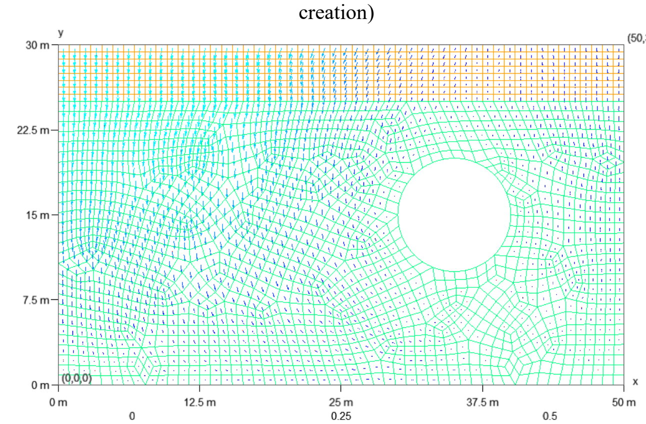

Fig. 14: Vertical displacement field (x5). With a rigid tunnel and settled work. (Source: Authors' own creation)

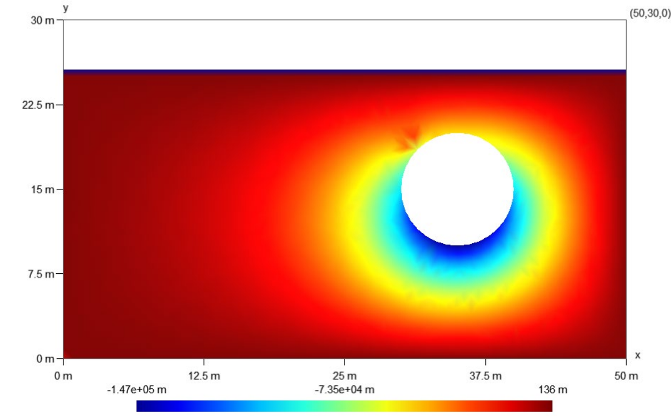

Fig. 15: Variation of interstitial pressures before and after the tunnel. (Source: Authors' own creation)

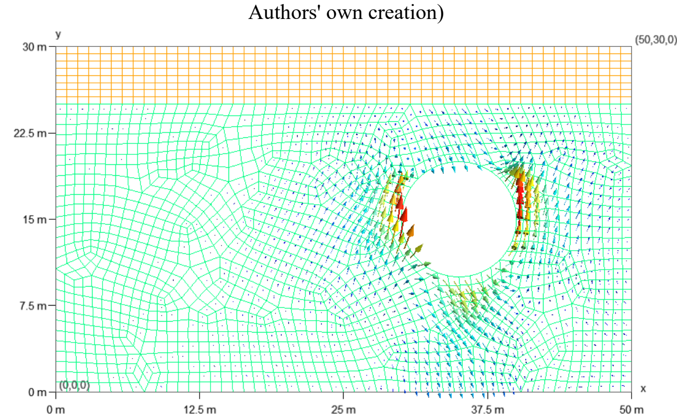

Fig. 16: Variation of effective stresses before and after the tunnel. Gradient. (Source: Authors' own creation)

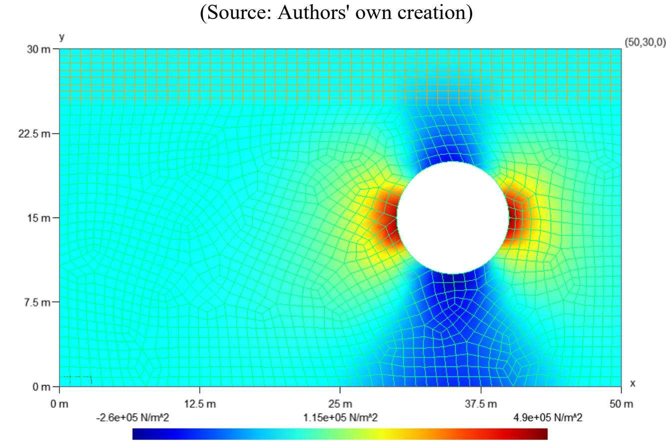

Fig. 17: Variation of effective stresses before and after the tunnel. (Source: Authors' own creation)$\hat{\sigma}_{yy}$

Once the total stresses have been obtained, the effective stresses are obtained by difference with the interstitial pressures. For $\hat{\sigma}_{xx}$:

Note comparing with Fig. 5 of total stresses and same case of permeable tunnel, as there are no changes in the first layer of soil (up to 5 m deep). This is because, since the soil is not saturated, since the water table does not invade it, the total and effective stresses coincide. The effect of interstitial pressures is appreciable in total stresses, both in *xx* and *yy*, as can be seen by comparing Figs. 5, 6, 8, 10 and 11.

However, the shear stress (Fig. 9), if we now compare the total tensions with the actual tensions, with and without tunnels, they do not differ between them. The figures are omitted because they are identical. This is because the *xy* components of effective stresses are not affected by interstitial pressures. This is reasonable considering that we are working with respect to effective stresses with an octahedral tensor, where hydrostatic thrusts can only be perpendicular, lacking xy components.

C. *Elastic settlement in accordance with Terzaghi's principle*

The approach that will be developed to resolve this section is based on the MEF, whose theoretical basis was discussed above. This is required in order to be able to obtain from his obtained results a correlation, which the genius of Terzaghi made known; That is, what has already been discussed, in other words: the deformation due to variation in the interstitial stresses depends, not so much on its magnitude as on the

TABLE III
ELASTIC SETTLEMENT DUE TO VARIATION IN EFFECTIVE STRESSES.

Key	**Screed**	**Foundations (x=10 m)**	**Foundations (x=35 m)**	**Incremental Differential Settlement (Terzaghi)**
4,00 ↓cm	4,30 cm ↑	≈ 0	1,00 cm ↓	≈ 1 cm ↓

TABLE IV
ELASTIC SETTLEMENT DUE TO VARIATION IN EFFECTIVE STRESSES.

Key	**Screed**	**Foundations (x=10 m)**	**Foundations (x=35 m)**	**Maximum differential settlement**
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

variation before and after the works that generate infiltration, in our case, of the construction of the tunnel. To solve the Terzaghi correlation, instead of using the stiffness matrix in the MEF, the volumetric strain modulus with the formulation already given above is used. By integrating the results matrices before and after the construction of the tunnel, in the simplified procedure followed, it is possible to estimate the variations that occur in the interstitial pressures and effective stresses in the keystone and floor. An analysis with a more complex procedure, but necessary for comprehensive contour analysis, is included in another work (Rodríguez *et al.,* 2023). Graphically, the results of the simplified procedure can be seen in Fig. 15.

The variation in interstitial pressures will participate in the greatest degree of variation in effective stresses, although it is not a constant relationship. They are as shown in Fig. 16 expressing the variation as a gradient. By affected areas, the variation in effective stresses can be seen in Fig. 17.

You can see in the figure the areas (cyan coloured) that have no appreciable variations in the effective stresses. A significant decompression is detected in the keystone and slab of the tunnel, which will lead to an increased risk of collapse. On the other hand, on the flanks there is a significant increase in effective stresses, which is also not favourable for the stability of the tunnel. Note that the unsaturated layer is not affected by the variation in effective pressures, except slightly in the area closest to the tunnel. The model with MEF under non-edometric conditions allows us to model this situation, in which the unsaturated wet soil is slightly sucked in by the saturated layer immediately below.

With Terzaghi's formulation, the incremental strain field is obtained due to the variation of the effective stresses. By integrating the field and selecting the nodes of interest, we have the seat produced by the variation of the effective stresses sought (Table 3). Since the settlement due to Terzaghi requires a variation in the effective stresses, if this settlement is considered on the surface, which is not affected by the flow problem, the settlement due to this cause will be negligible. However, on the safety side and for the purpose of estimating the settlement due to Terzaghi, without considering ascents of the water table to the surface, a vertical and downward displacement has been considered referring to the nodes located under the foundation at a depth of 6 m. This assumption, on the side of safety for the case at hand, should be limited to an extended area of influence of the area affected by the flow problem. Proceeding in this way for the fixed value of 6 m, the results of tables 3 and 4 are obtained.

Approximately 35% of the seats in the tunnel key due to infiltrations. In addition, the model detects a strong decompression in that area. The differential settlement of the building area is increased by around 23%. Therefore, the effect of tunnel infiltrations cannot be underestimated in the problem analysed. The total seat produced (Table 4), with the presence of infiltrations in the tunnel in a granular medium of the case analysed, is the instantaneous seat plus the primary consolidation seat. According to the numerical model used, the primary consolidation seat is around 12% on the upper surface, without direct contact with the water table but affected by the loss of effective stresses in the ground below it, through an area of influence. The model allows this effect to be assessed. It is foreseeable that this settlement will occur in a very short primary consolidation time due to the high permeability of the soil. As for secondary consolidation by creep, negligible in completely granular soils, in accordance with the CTE-SE-DBSE-C (Technical Building Code, 2019) standard and knowing that the soils are predominantly granular, but in practice are rarely free of fines, on the safety side it could be estimated at a maximum of 20% of the seats indicated in table 4.

IV. CONCLUSIONS

The variation in effective pressures produces a seat that is added to the elastic seat itself. In the case study, the construction of the tunnel entails an increase in the elastic differential settlement of the building area of around 23% considering infiltrations in the tunnel.

The variation in effective pressures induces the failure of the tunnel in its screed. The tunnel "rises", generating an additional risk to the collapse due to a possible failure in the support of the vault.

It is checked that the effective stresses are not affected by the shear stress. This is consistent with the characteristics of the flow problem in porous medium. It is found that the volumetric deformations derived from the variation of the effective pressures are only affected by the octahedral effective stresses.

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