Diagramas de interacción axil-flector en secciones doble "T". Empleo de programa Matlab en aplicaciones estructurales a nivel de sección (parte I).
Axial-bending interaction diagrams for "I" sections. Structural applications related to section study using Matlab (part I).

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Resumen—El método multicapa es una potente herramienta de cálculo en el campo de la ingeniería para tratar problemas a nivel de sección. Este método permite tratar problemas relacionados con el comportamiento no-lineal de los materiales, el empleo de materiales con diferentes propiedades en una misma sección, o problemas relacionados con gradiente de temperatura, entre otros. Dos aplicaciones serán tratadas en dos artículos independientes. En este documento el método multicapa se empleará para generar un diagrama de interacción axil-momento para secciones tipo doble "T". La sección de acero se estudiará respecto al régimen elástico y plástico, y se propondrá un parámetro que aportará información sobre la reserva plástica de la sección, cuando actúan simultáneamente un axil y un momento flector, la denominada área plástica. Se ha empleado como entorno de programación el software Matlab para desarrollar el contenido de este artículo.

Palabras clave— Método multicapa; iteración; diagrama; plasticidad; acero; matlab.

Abstract- Multi-layer method is a powerful tool to solve structural engineering problems from a section point of view. This method can solve problems related to non-linear material behavior, different materials at the same section, or aspects related to temperature gradient, among others. Two applications shall be explained in two separate technical papers. In this paper multi-layer method shall be used to generate an axial-bending interaction diagram for "I" steel sections. The section shall be studied from elastic and plastic material properties and a parameter shall be proposed (plastic area) in order to obtain the plastic capacity of a section produced by axial forces and bending moment acting simultaneously. Matlab software had been employed to develop the content of this technical paper.

Index Terms— Multi-layer method; iteration; diagram; plasticity; steel; matlab.

I. INTRODUCTION

Frequently it is more usual to find in technical papers, books or any documentation prepared in colleges or research centres, the results, calculations, numerical developments or graphics generated by software tool Matlab. Matlab is a powerful programing environment which includes thousands of predefined mathematical commands. It represents a great advantage in order to solve a wide range of engineering problems, without a deep programing knowledge from the user point of view.

In the field of structural engineering, there are a several methods to study the structural behavior of a section. One of these methods is the "Mutli-Layer Method" which shall be

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adopted in this technical paper. Matlab shall demonstrate the structural behavior of a section through this method, by employing two types of materials: steel and reinforced concrete. The analysis of each materials shall be discussed into two separate articles (Part I Steel and Part II Reinforced Concrete).
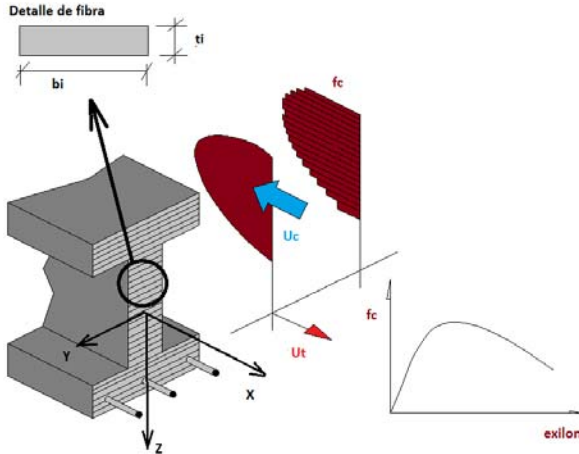

Fig. 1. Descriptive Sketch for "Multi-Layer Method". Stress-strain model, dimensions of a general layer, axis reference together with forces are all represented. Among the possibilities of the method, non-linear material behavior treatment is one of the most significant.

In part I of this article, which is discussed in succeeding pages, an interaction axial-bending diagram for "I" steel section types shall be developed first. This matter shall be dealt under an elastic and plastic limit states. On the latter part of this article it shall be proposed the parameter called "plastic area" which tries to provide information about the plastic capacity of a single section subjected simultaneously to axial force and bending moment.

In the part II, which is discussed in a separate article, curvature value associated with first yielding of steel reinforcement shall be obtained. This parameter is extremely important in seismic engineering design, since together with curvature associated with ultimate moment capacity, section ductility parameter can be defined. For this purpose, stress-strain models of concrete material (Mander model) and rebar material (King model) shall be used, which shall allow to study and analyze the problem from a non-linear material behavior.

In both articles two applications shall be explained technically in detail, and by means of a worked examples and the results provided by Matlab, some conclusions can be drawn. Programing codes developed in Matlab shall be shown in the appendix of the articles for benefit of the readers.

II. MULTI-LAYER METHOD

Basically the Multi-layer method is done by dividing the section in a certain number of layers or "slices". Each layer can exhibit the deformation state versus an external force applied, and therefore the stress levels, whatever stress-strain material model is assumed. Among the advantages of the method are the following: non-linear material application, different material properties acting at the same section, aspects related to temperature gradient applied to a section or cracking in concrete sections, among others.

In case that the forces and bending moments are generated due to normal stresses in longitudinal direction of the element (see figure 1 X direction), the value of force and bending moment shall be calculated by adding the whole contributions of each layer, since this process represents the concept of stress integration all along the section depth. Needless to say, as more number of layer defined the higher accuracy of the results can be produced.

Axil force

Continuous way $\qquad N_X(x) = \int_0^A \sigma x(x, y, z) dA$

Discrete way $\qquad N_x(x) = \sum_{i=1}^i \sigma x_i * t_i * b_i$ (1)

Bending Moment

Continuous way $\qquad M_y(x) = \int_0^A z * \sigma_x(x, y, z) dA$

Discrete way $\qquad M_y(x) = \sum_{i=1}^i z * \sigma x_i * t_i * b_i$

III. AXIAL-BENDING INTERATION DIAGRAM FOR "I" STEEL SECTIONS

Most international steel codes allow to design different elements adopting a plastic limit state, as long as some geometrical and material conditions have been met. According to Ec-3 part 1-1, section 5.5, a section can be defined as compact and therefore section plastic design as suitable, if the following conditions have been fulfilled:

Web $\qquad c/t \leq 396\varepsilon/(13\alpha - 1)$ for $\alpha{>}0.5$

Flanges $\qquad c/t \leq 9\varepsilon$

Where ε is defined as:

$$\varepsilon = \sqrt{\frac{235}{f_y}}$$

Dimensions "t" represents the specified thickness of the section part, and "c" is the height of the web or the length of half flange. For further information refer to table 5.2 of the aforementioned code.

A value equal to 0.82 is adopted conservatively for alpha parameter, since the web is subjected to compressive force and bending moment simultaneously.

The advantages of plastic limit design as compared to elastic limit design is mainly bounded by the economic point

of view of the material and a relationship in between section design and plastic analysis can be established directly. When a section is subjected exclusively to bending moment the variable which provides information about plastic capacity is the one called Shape Factor (f), and that is equal to the ratio between plastic and elastic modulus. Shape factor is not related to the grade of the material.
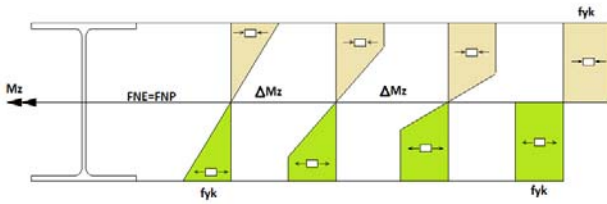
$$f = \frac{Mpl}{Mel} = \frac{Wpl}{Wel}$$

Fig. 2. Normal stress diagrams due to bending moment applied in strong axis. Compact sections are those sections which can form a plastic hinge, without the reduction of capacity due to local buckling. These sections are defined in Ec-3 as Class 1 and 2.

In case that a section is subjected to axial forces and bending moments acting at the same time, an interaction diagram can be generated that represents its structural behavior assuming the plastic limit state. The gradual yielding of the section is made available by the "Multi-Layer Method", which under this circumstances few layers shall be dealing with axial stresses and the rest shall be dealing with flexural stresses. In figure 3, the procedure is explained graphically.
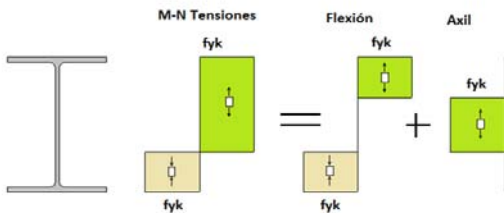

Fig. 3. Multi-Layer Method application. External fibers deals with the flexural capacity meanwhile internal core fibers are dealing with the axial capacity.

It is not necessary to mention the utility of an interaction diagram from a design point of view, but this representation cannot be limited to that extent alone. It is also possible to establish a parameter to measure the extra plastic capacity for a section if it is compared against elastic limit state. The plastic area parameter can be obtained by following the steps listed below:

1) Section shall be compact according to the geometrical and material specifications in accordance with the reference code employed, otherwise plastic design criteria cannot be adopted.

2) Calculate ultimate bending moment and axial force under elastic limit state. By graphical solution, interaction diagram is derived, by straight line defined by two points only.

3) By means of Multi-Layer Method, a pair of values corresponding with axial force and bending moment shall be generated, by gradually yielding ofeach fiber, as described in figure 3. The more pair of values obtained, the more accuracy can be attained in generating the plastic interaction diagram.

4) Once both interaction diagrams are created, these shall be represented in non-dimensional way, by dividing every single value by the ultimate values of plastic and axial bending respectively.

5) Non-dimensional interaction diagrams shall be overlapped. The difference between those enclosed areas resulted into the parameter called in this technical paper "Plastic Area" (see figure 4). Dealing with non-dimensional data make it easier to derive results and therefore to make comparison with other section to establish which has the best plastic capacity.

$$A_{plástica} = A_{N-Mp} - A_{N-Me}$$
$$A_{N-Mp} = \int_0^1 f\left(\frac{N}{Np}\right) d\left(\frac{N}{Np}\right)$$
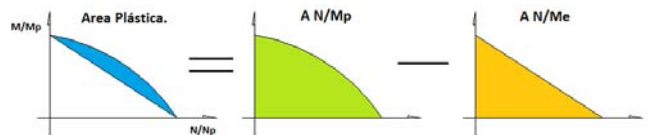$$A_{N-Me} = \frac{1}{2} * \frac{M}{Mp} * 1$$

Fig. 4. Determination of "Plastic Area" as result of subtraction between plastic interaction diagram and elastic interaction diagram.

IV. WORKED EXAMPLE

It is required to obtain the "Plastic Area" parameter for a built-up steel section with dimensions specified in figure 5, with a yield limit of 355 Mpa, and assuming an elastic-perfectly plastic model. In order to avoid the strength reduction due to instability problems, member is assumed to be subjected to tensile axial force exclusively and is laterally restrained.
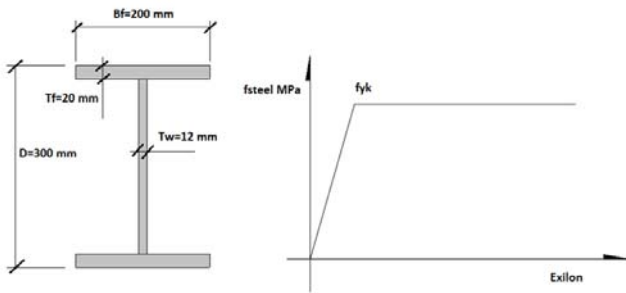
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Fig. 5. Built-up section dimensions and stress-strain model adopted.

1) Compact Section checking.
The requirements of compact sections shall be fulfilled as specified by Ec-3 as:

Web $\qquad c/t \le \frac{396\varepsilon}{13\alpha-1} \qquad 21.67 \le 33.35 \qquad Ok$

Flanges $\qquad c/t \le 9\varepsilon \qquad\quad 4.70 \le 7.312 \qquad Ok$

$$\varepsilon = 0.81$$

2) Axial and bending values under elastic limit state.

$$N_{pl} = A * f_{yk} = 3947 \, KN$$
$$M_{el} = Wel * f_{yk} = 413.32 \, mKN$$
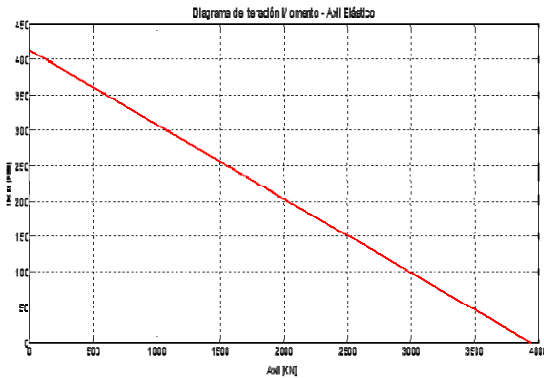$$\frac{M}{M_{el}} + \frac{N}{N_{pl}} = 1$$

Fig. 6. Interaction axil force and bending moment Interaction diagram under elastic limit state. Units in KN and KNm.

3) Axial force and bending moment values under plastic limit state.

Multi-layer method shall provide the axial and bending points which generate the interaction diagram in the plastic range. In Y-axis the ultimate plastic moment can be found and ultimate axial force in X-axis. These values and the shape factor are shown below:

$$N_{pl} = A * f_{yk} = 3947 \, KN$$
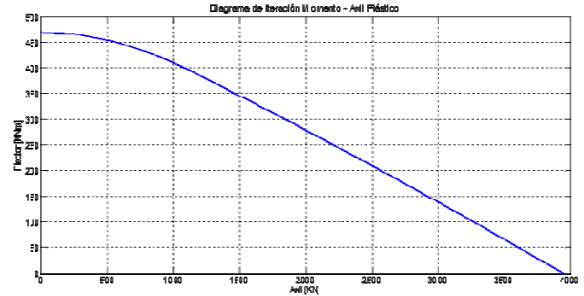$$M_{pl} = Wpl * f_{yk} = 465.09 \, mKN$$

$$f = \frac{Mpl}{Mel} = 1.13$$

Fig. 7. Axial force and bending moment Interaction diagram under plastic limit state. Units in KN and KNm.

4) Non-dimensional interactions diagrams.
Dividing every single point by ultimate axial and ultimate plastic value respectively, the non-dimensional diagrams can be achieved. In the case of the elastic diagram the equation that described the results is the following:

$$\frac{M}{M_{el}} + \frac{N}{N_{pl}} = 1$$
$$N = N' * N_{pl} \quad N' = [0,1]$$
$$M = M' * M_{pl} \quad M' = [0, M_{el}/M_{pl}]$$
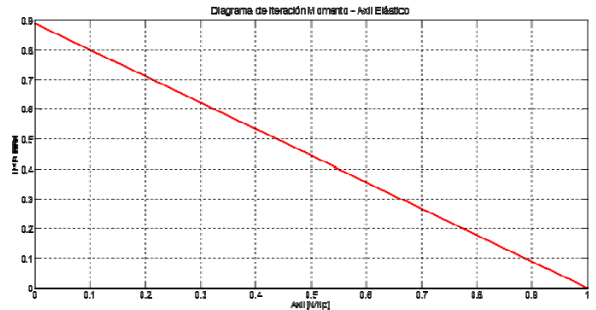$$\frac{M' * M_{pl}}{M_{el}} + N' = 1$$

Fig. 8. Non-dimensional axial force and bending moment Interaction diagram under elastic limit state.
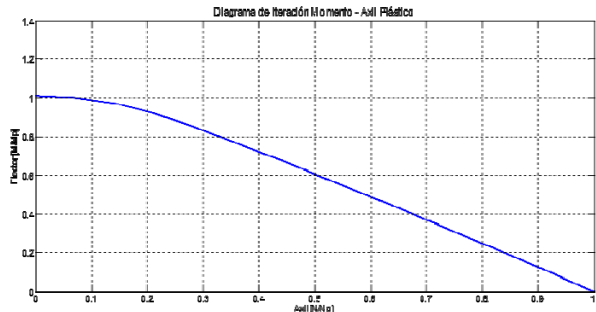

Fig. 9. Non-dimensional axial force and bending moment Interaction diagram under plastic limit state.

As shown on the diagram represented in figure 9, the curve

is practically linear up to the axial point equal to 0.2. This aspect matches with technical reference consulted, and the reason can be attributed to little contribution of flanges to overall sectional flexural capacity.

Also in figure 9 another conclusion can be drawn and the explanation of why codes does not considered in flexural design values of axial forces with a magnitude less than 10% of the ultimate axial capacity. As it can be seen there is no practical reduction of the bending strength capacity associated with axial magnitudes below 10%.

5) *Overlapped Interaction diagrams.*
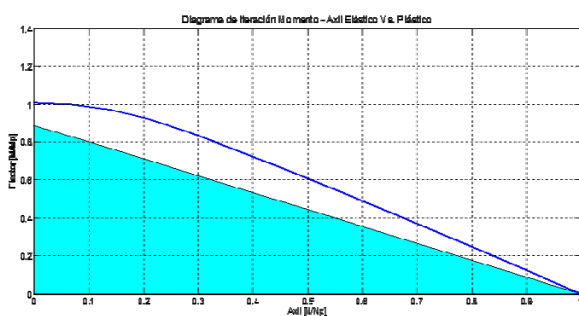The consequence of overlapping the two diagrams is represented in figure 10.

Fig. 10. Overlapping Non-dimensional axial force bending moment Interaction diagrams.

In this particular case the value of the plastic area is 0.1379.

V. CONCLUSIONS

As demonstrated in this article the matlab programing software can be utilized to implement the Multi-Layer Method, and then providing good structural engineering solutions. It is also demonstrated in this article the advantages of the generation of the axial force-bending moment interaction programs for "I" section in plastic limit state, as a tool to obtained the plastic capacity when axil and bending acting simultaneously, by mean of the parameter "Plastic Area".

APPENDIX

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%    MULTI LAYER METHOD FOR INTERACTION DIAGRAM
%       UNDER PLASTIC AND ELASTIC CRITERIA
%         IN "I" SIMETRICAL SECTIONS
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Author: Sergio Rodríguez Morales
%Date: 21th February 2015
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%El programa cálculo mediante un modelo multicapa la capacidad
%resistente plástica axil y flectora de una sección en doble T.
%El programa irá plastificando cada uno de las fibras definidas
%empezando por asignar toda la capacidad de las fibras a la capa-
%cidad a flexión de la sección, para posteriormente ir asignando
%fibras plastificadas a la capacidad resistente frente a axil.
%
%Unidades: Todo en kp y cm, aunque la salida se representa en KN y
%KN*m. De cara al exterior modernidad pero los principios clásicos.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
close all
```

```
clear

%1. DATA INPUT
h= 30      % DEPTH SECTIONcm
bf= 20     % WIDTH SECTION cm
tw=1.2     % WEB THK cm
tf=2       % FLENAGE THK cm
fyk=3550   % STEEL YIELD LIMIT kp/cm^2

nfala=20   % NUMBER OF LAYERS IN ONE FLANGE
nfalma=20  % NUMBER OF LAYERS IN THE HALF WEB

%2.OPERATIONS

%2.1. PLASTIC ANALYSIS
fala=tf/nfala              % FLANGE THK FOR EACH LAYER
falma=(h/2-tf)/nfalma      % WEB THK FOR EACH LAYER

n=nfala+nfalma             % TOTAL NUMBER OF LAYERS

diagrama_flector=zeros(n+1,1)
%COLUMN VECTOR OF ZEROS AND FIRST VALUE M=0 BENDING
diagrama_axil=zeros(n+1,1)
%COLUMN VECTOR OF ZEROS AND FIRST VALUE N=0 AXIAL

%AXIAL CAPACITY FOR EACH LAYER.
%RESULTS STORED IN VECTOR axilfibra
for i=1:n
    if i<=nfala
        axilfibra(i)=fala*bf*fyk
    end
    if i>nfala
        axilfibra(i)=falma*tw*fyk
    end
end

%DISTANCE FROM COG TO EACH LAYER
%RESULTS STORED IN VECTOR distfibra
distfibra=zeros(n+1,1)
%DISTANCE TO FLANGES LAYERS.
for i=1:(nfala)
    distfibra(i)=h/2-(2*i-1)*fala/2
end
%DISTANCE TO WEB LAYERS
for i=1:nfalma
    distfibra(i+nfala)=(h/2-tf)-(2*i-1)*falma/2
end

%LOOPING FOR CALCULATE AXIAL AND BENDING FOR EACH LAYER

axil_acum=0
flector_acum=0

%TOTAL BENDING MOMENTS GENERATION
for i=1:n
%     if i==1
%     diagrama_flector(i,1)=0
%     end
    flector_acum=2*distfibra(i,1)*axilfibra(1,i)+flector_acum
    diagrama_flector(i+1,1)=flector_acum/1e4
end
%TOTAL AXIAL CAPACITY OF THE SECTION
for i=1:n
    axil_acum=axilfibra(1,i)+axil_acum
end
%ONLY HALF SECTION IS COMPUTED THE FINAL RESULT SHALL BE DOUBLE
axil_acum=2*axil_acum
%GENERATION OF AXIAL ACTING SIMULTANEASLY WITH BENDING
for i=1:n
    if i==1
        diagrama_axil(i,1)=axil_acum/100
    end
    if i>1
    axil_acum=axil_acum-2*axilfibra(1,i-1)
    diagrama_axil(i,1)=axil_acum/100

    end
end

%2.2 ELASTIC ANALYSIS
% STRENGTH AND MECHANICAL PROPERTIES
h1=h/2-tf                        %Half web height
A=2*bf*tf+(h-2*tf)*tw
Ix=2*(bf/3*(h^3/8)-h1^3*(bf-tw)/3)   %Second moment of inertia
Wx=Ix/(0.5*h)
Mel=fyk*Wx/1e4                   %Elastic bending moment
Nu=fyk*A/100                     %Axial capacity

%3. OUTOUT

%PLASTIC AXIAL-BENDING INTERACTION DIAGRAM.
figure
area(diagrama_axil,diagrama_flector,'Facecolor','c')
plot(diagrama_axil,diagrama_flector)
grid on
title('Diagrama de iteración Momento - Axil Plástico','Fontsize',16)
xlabel('Axil [KN]')
ylabel('Flector [KNm]')


%ELASTIC AXIAL-BENDING INTERACTION DIAGRAM.
```

```
figure
y=[0,Mel]
x=[Nu,0]
plot(x,y)
grid on
title('Diagrama de iteración Momento - Axil Elástico','Fontsize',16)
xlabel('Axil [KN]')
ylabel('Flector [KNm]')

% PLASTIC-ELASTIC OVERLAPPING DIAGRAMS
figure
y=[0,Mel]
x=[Nu,0]
area(x,y,'FaceColor','c')
hold on
plot(diagrama_axil,diagrama_flector,'b')
grid on
title('Diagrama de iteración Momento - Axil Elástico Vs.
Plástico','Fontsize',16)
xlabel('Axil [KN]')
ylabel('Flector [KNm]')

% PLASTIC-ELASTIC OVERLAPPING DIAGRAMS NON-DIMENSIONAL
figure
diagrama_axil_adi=diagrama_axil/Nu
diagrama_flector_adi=diagrama_flector/diagrama_flector(n+1,1)
plot(diagrama_axil_adi,diagrama_flector_adi,'b')
hold on
y=[0,Mel/diagrama_flector(n+1,1)]
x=[1,0]
plot(x,y,'r')
grid on
title('Diagrama de iteración Momento - Axil Elástico Vs.
Plástico','Fontsize',16)
xlabel('Axil [adim]')
ylabel('Flector [adim]')

% PLASTIC AREA, DIFERENCE BETWEEN PLASTIC AND ELASTIC

%Area UNDER STRAIGHT LINE
area_recta=0.5*Mel/(diagrama_flector(n+1,1))*Nu/Nu
%AREA UNDER PLASTIC CURVE
area_curva=trapz(abs(diagrama_axil_adi),abs(diagrama_flector_adi))
disp('el area y por tanto la reserva plastica del perfil es_:')
area_plastica=abs(area_curva+area_recta)
```

VI. ACKNOWLEDGMENT

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